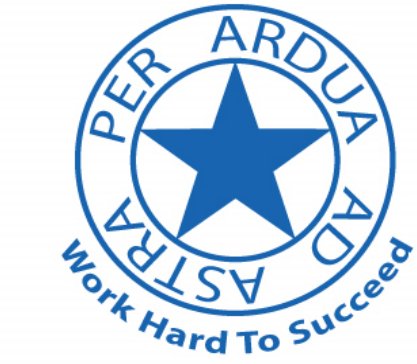
Garden Suburb Junior School



Calculations Policy

Introduction

This calculations policy has been written to provide an understanding of when and how the four operations (addition, subtraction, multiplication and division) are taught at Garden Suburb Junior School.

The aim of this policy is to ensure consistency throughout the school and to make teachers and parents aware of the continuity and progression in skill development across the year groups in addition, subtraction, multiplication and division. It aims to enable staff, and parents, to see how the concepts, facts and calculation strategies and methods used are taught, and how these build on previous learning and contribute to future learning.

It also aims to emphasise the important link between the development of children's mental calculation strategies and the teaching of written calculation methods. Mental recall and strategies, and formal written methods must be seen as complementary to each other. Every written method has a component of mental processing so the two must constantly be developed in conjunction with each other. Pupils' mental facilities with number should be refined as they move through Key stage 2 and not focus exclusively on the written methods of calculation.

Although this Calculations Policy is organised by operation it is also vital to stress the importance of the interconnectivity between the four operations; pupils' conceptual understanding; fluency in number facts; mental strategies and written methods, and their ability to reason mathematically and solve problems.

Our ultimate goal is to ensure that children are confident and competent in their calculation skills, and are able to use and apply these skills in the real world as autonomous problem solvers.

We believe that being able to calculate successfully is being able to:

- have a confident and competent understanding of numbers and the number system
- have instant recall of a set of basic number facts
- use a range of mental calculation strategies effectively, efficiently and flexibly
- use a range of written calculation methods accurately and appropriately
- use and apply all of the above in order to solve problems and reason mathematically

The structure of this document:

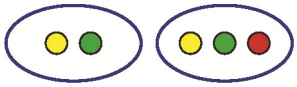
The document is divided into the four operations: addition, subtraction, multiplication and division. For each operation a progression in skills for the mental calculation strategies taught at Garden Suburb Junior School is explained. This is followed by an explanation of the progression in written methods taught exemplifying the informal and formal methods.

The informal written methods taught lead to the formal written methods so that children fully understand the procedure, and the effectiveness and efficiency of the method. The amount of time that should be spent teaching and practising the informal written methods will depend on how secure the children are in their recall of + - x ÷ number facts and in their understanding of place value. This will therefore vary from child to child depending on their needs.

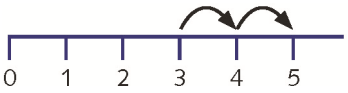
Addition

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations e.g.



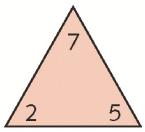
  - number tracks and number lines e.g.



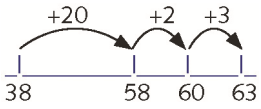
  - 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

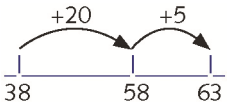
  - trios e.g. 5 + 2 = 7
    2 + 5 = 7
    7 – 2 = 5
    7 – 5 = 2



  - empty number line e.g.



leading to:



- Recognise and use the inverse relationship between addition and subtraction
- Use knowledge that addition can be done in any order (commutative), e.g.
  - put the larger number first and count on in steps of 1, 10, 100 or 1000
  - add three small numbers by putting the largest number first and/or find a pair totalling 10
- Partition additions then recombine e.g.
  - 75 + 56 = 75 + 50 + 6
  - 356 + 57 = 356 + 50 + 7
  - 3356 + 257 = 3356 + 200 + 50 + 7
- Identify near doubles, using doubles already known, e.g.
  - 70 + 71
  - 170 + 180
  - 5·7 + 5·8

- Add the nearest multiple of 10, 100 or 1000, and adjust e.g.
  - 30 + 48 = 30 + 50 - 2
  - 680 + 400 = 680 + 500 - 1

- Use patterns of similar calculations, e.g.
  - 13 + 5 = 18 and 130 + 50 = 180
  - 9 + 7 = 16 and 0·09 + 0·07 = 0·16

- Use knowledge of the associative law when adding more than two numbers, e.g.
  4 + 7 + 6 = (4 + 6) + 7 = 10 + 7= 17
  24 + 27 + 16 = (24 + 16) + 27= 40 + 27= 67

Addition

Written methods

- Estimate and check the answer to a calculation

Informal Method

Expanded written method

38 + 25

```
  38
+ 25
  13
  50
  63
```

548+387

```
  548
+ 387
   15
  120
  800
  935
```

Record addition calculations in columns to support place value and prepare for the formal written method of columnar addition with larger numbers.

The first stage in the written method shows separately the addition of the ones to the ones and the tens to the tens. To find the partial sums either the ones or the tens can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method. The addition of the tens in the calculation 38 + 25 is described in the words ‘thirty add twenty equals fifty’, stressing the link to the related fact ‘three add two equals five’.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

Formal written method of columnar addition

```
  1 1
  548
+ 387
  935
```

```
    1 1
  2 4 5 6
+ 5 3 7 8
  7 8 3 4
```

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Carry digits are recorded above the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labeled, e.g. HTO, to remind children of the value of each of the digits.

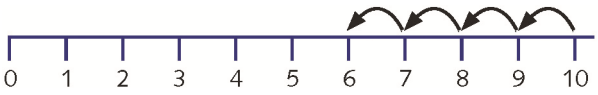
If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

Subtraction

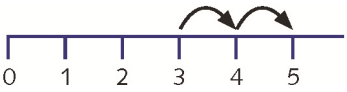
Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations e.g.

  - number tracks and number lines: 'take away' (counting back) e.g.



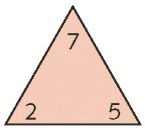
  - 'finding the difference' (counting up) e.g.



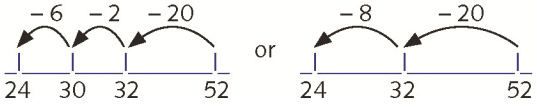
  - 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

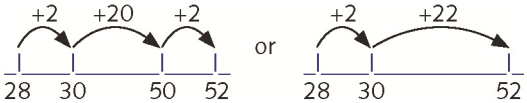
  - trios e.g. 5 + 2 = 7
    2 + 5 = 7
    7 – 2 = 5
    7 – 5 = 2



  - empty number line: 'take away' (counting back) e.g.



  - 'finding the difference' (counting up) e.g.



- Find a small difference by counting up from the smaller to the larger number, e.g.
  - 29 – 27 count up from 27 to 29

- 700 – 658 count up from 658 to 700

- Subtract the nearest multiple of 10, 100 or 1000, and adjust e.g.
  - 56 - 29 = 56 – 30 + 1
  - 6783 – 3800 = 6783 – 4000 + 200

- Understand and use the inverse relationship between addition and subtraction

- Use partitioning, e.g.
  - 52 – 28 = 52 – 20 – 8
  - 456 – 84 = 456 – 80 – 4
  - 4656 – 358 = 4656 – 300 – 50 – 8

- Use patterns of similar calculations, e.g.
  - 18 – 5 = 13 and 180 – 50 = 130
  - 16 – 9 = 7 and 1·6 – 0·9 = 0·7

Subtraction

Written methods

- Estimate and check the answer to a calculation

- Subtract two two-digit numbers: TO – TO (that do not require decomposition)
  87 – 32

$$\begin{array}{r} 87 \\ -\ 32 \\ \hline 55 \end{array}$$

Record subtraction calculations that do not require decomposition in columns to support place value and prepare for formal written methods of columnar subtraction with larger numbers.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

Formal written method of columnar subtraction (decomposition)

273 – 165 83·72 – 36·49

$$\begin{array}{r} {}^{6} \\ 2\,\not{7}^{1}3 \\ -\ 1\,6\,5 \\ \hline 1\,0\,8 \end{array} \qquad \begin{array}{r} {}^{7}\ {}^{6} \\ {}^{1}\not{8}\,3\cdot\not{7}^{1}2 \\ -\ 3\,6\cdot 4\,9 \\ \hline 4\,7\cdot 2\,3 \end{array}$$

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying ‘14 tens subtract 8 tens’, not ‘14 subtract 8’.

In the first example, the calculation begins by exchanging one of the 7 tens for 10 ones, crossing out the 7 and writing a superscript 6. The calculation then becomes 13 subtract 5.

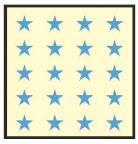
Where appropriate, place value columns are labelled, e.g. TO·th, to remind children of the value of each of the digits.

Multiplication

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations e.g. 4 x 5 =

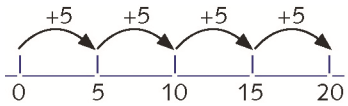
  - arrays

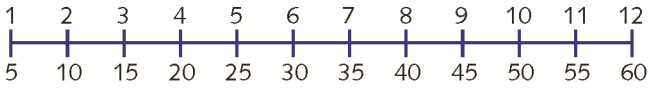


4 × 5 = 20

5 × 4 = 20

  - number lines





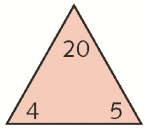
  - trios

4 × 5 = 20

5 × 4 = 20

20 ÷ 5 = 4

20 ÷ 4 = 5



- Make connections between arrays, number patterns and counting in steps of a constant size

- Understand and use the inverse relationship between multiplication and division, including doubling and halving

- Use doubling, e.g.
  - connect the 2, 4 and 8 multiplication tables
  - double or halve the most significant digit first
  - to multiply by 25, multiply by 100 then divide by 4
  - double one number and halve the other
  - find the multiplication facts for the × 16 multiplication table by doubling the × 8 multiplication facts
  - find the multiplication facts for the × 24 multiplication table by doubling the × 12 multiplication facts
  - squares of multiples of 10 to 100, e.g. 70 × 70, and the corresponding halves
  - doubles of decimals, e.g. 4·7 × 2, 0·63 × 2, and the corresponding halves
  - doubles of multiples of 10 to 1000, e.g. 830 × 2, and the corresponding halves
  - doubles of multiples of 100 to 10 000, e.g. 48 500 × 2, and the corresponding halves

- Use the 'key multiplication facts' of × 1, × 2, × 5, and × 10 to work out the answers to unknown multiplication facts,

  e.g. 7 × 9 = (5 × 9) + (2 × 9)

  = 45 + 18

  = 63

- Use closely related facts:
  - multiply by 9 or 11 by multiplying by 10 and adjusting
  - develop the × 12 table by adding facts from the × 10 and × 2 table

- Use factors, e.g. 8 x 14 = 8 × 2 × 7

- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 8 × 60 = 480

- Understand and use the commutative law, e.g.

  14 × 12 = (2 × 7) × 12
  = 2 × (7 × 12)
  = 2 × 84
  = 168

- Understand and use the associative law, e.g.

  6 × 15 = 6 × (5 × 3)
  = (6 × 5) × 3
  = 30 × 3
  = 90

- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, e.g.

  356 × 7 = (300 × 7) + (50 × 7) + (6 × 7)
  = 2100 + 350 + 42
  = 2492

Multiplication

Written methods

- Estimate and check the answer to a calculation

Informal Written Methods – Short Multiplication

Grid method

63 × 8

×	60	3
8	480	24

= 504

Expanded written method

63 × 8

$$\begin{array}{rl} 6\ 3 & \\ \times\quad 8 & \\ \hline 2\ 4 & (3 \times 8) \\ 4\ 8\ 0 & (60 \times 8) \\ \hline 5\ 0\ 4 & \\ {\scriptstyle 1} & \end{array}$$

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 63 × 8 it is 'sixty multiplied by eight', not 'six multiplied by eight', although the relationship 6 × 8 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

$$\begin{array}{r} {\scriptstyle 2} \\ 6\ 3 \\ \times\quad 8 \\ \hline 5\ 0\ 4 \\ \hline \end{array}$$

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Informal Written Methods – Long Multiplication

Grid method

78 × 34

×	70	8	
30	2100	240	2340 →
4	280	32	+ 312 →
			2652 →

$$\begin{array}{rll} 7\ 8 & & \\ \times\quad 3\ 4 & & \\ \hline 2\ 3\ 4\ 0 & (78 \times 30) & \\ 3\ 1\ 2 & (78 \times 4) & \\ \hline 2\ 6\ 5\ 2 & & \end{array}$$

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 78 is multiplied by 30 (using knowledge of 78 x 3), then 78 is multiplied by 4, and finally the two products are added together.

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Formal written method of long multiplication

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 24 \\ \times\ 16 \\ \hline 240 \\ 144 \\ \hline 384 \\ \hline \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 124 \\ \times\ 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ {\scriptstyle 1\ 1} \end{array}$$

Answer: 3224

Children can choose to multiply by the most significant digit first, as shown in the first and second example, or to multiply by the least significant digit first, as shown in the third example. These are recorded below in the appropriate columns and then added to complete the calculation.

Long multiplication (Decimals):

Method 1: Calculating with decimals

Grid method

7·56 × 34

×	7	0·5	0·06
30	210	15	1·8
4	28	2	0·24

$$\begin{array}{r} {\scriptstyle 1} \\ 226{\cdot}80 \\ +\ \ 30{\cdot}24 \\ \hline 257{\cdot}04 \end{array}$$

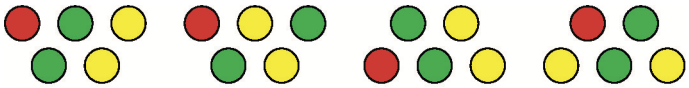
The first step is to show all of the calculations involved, e.g. 7·56 is multiplied by 30 (using knowledge of 756 x 3), then 7·56 is multiplied by 4, and finally the two products are added together.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

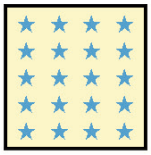
Division

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations e.g. 20÷5

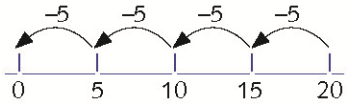


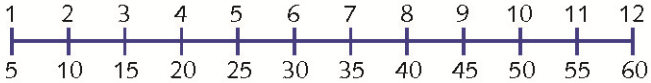
  - arrays e.g



20 ÷ 5 = 4
20 ÷ 4 = 5

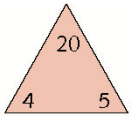
  - number lines e.g.





  - trios e.g.

4 × 5 = 20
5 × 4 = 20
20 ÷ 5 = 4
20 ÷ 4 = 5



- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division, including doubling and halving
- Make connections between arrays, number patterns and counting in steps of a constant size
- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number e.g.
  486 ÷ 9 = (450 ÷ 9) + (36 ÷ 9)

Division

Written methods

- Estimate and check the answer to a calculation

Informal Method

Expanded written method

$92 \div 4$

$$\begin{array}{r|l} 4\overline{)92} & \\ 80 & 20 \times 4 \\ \hline 12 & \\ 12 & 3 \times 4 \\ \hline 0 & \end{array}$$

$486 \div 9$

$$\begin{array}{r|l} 9\overline{)486} & \\ 450 & 50 \times 9 \\ \hline 36 & \\ 36 & 4 \times 9 \\ \hline 0 & \end{array}$$

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answer is a multiple of 10? (20) There are 20 fours or 80, with 12 remaining. How many fours in 12? (3) So 92 divided by four is 23.'

Formal written method of short division

$$4\overline{)9\,{}^{1}2} \qquad 23$$

$$9\overline{)4\,8\,{}^{3}6} \qquad 54$$

The expanded written method leads children to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90. It is written in front of the 2 to show that 12 now has to be divided by 4.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Formal written method of short division with remainders

Whole number remainder

$279 \div 6$

$$6\overline{)2\,7\,{}^{3}9} \qquad 46\text{r}3$$

Fraction remainder

$279 \div 6$

$$6\overline{)2\,7\,{}^{3}9} \qquad 46\tfrac{1}{2}$$

Decimal remainder

$279 \div 6$

$$6\overline{)2\,7\,{}^{3}9\cdot{}^{3}0} \qquad 46\cdot5$$

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10? (40) There are 40 sixes or 240, with 30 remaining.' The superscript 3 represents the 3 tens that are remaining after 6 has been divided into 270. It is written in front of the 9 to show that 39 now has to be divided by 6.

Children then ask: 'How many sixes in 39?' (6 remainder 3). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

Dividing with decimals

Method 1: Calculating with decimals

$$6\overline{)45\cdot{}^{3}3\,{}^{3}6} \qquad 7\cdot56$$

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

45·36 ÷ 6 is equivalent to 4536 ÷ 6 ÷ 100

$$\begin{array}{r} 7\,5\,6 \\ 6\overline{)\,4\,5^{3}3^{3}6} \end{array}$$

756 ÷ 100 = 7·56

Phrases similar to those above for short division of whole numbers should be used for short division involving decimals.

An emphasis should be placed on recognising the value of each of the digits in the dividend.

Long Division

Expanded written method of long division

$$\begin{array}{rl} 3\,2\,4\ \mathrm{r}\ 4 & \\ 18\overline{)\,5\,8\,3\,6} & \\ -\ 5\,4\,0\,0 & (300 \times 18) \\ \hline {}^{3}4\,{}^{13}3\,6 & \\ -\ 3\,6\,0 & (20 \times 18) \\ \hline 7\,6 & \\ -\ 7\,2 & (4 \times 18) \\ \hline 4 & \end{array}$$

5836 ÷ 18 = 324 r 4 or $324\frac{2}{9}$

Formal written method of long division

$$\begin{array}{r} 3\,2\,4\ \mathrm{r}\ 4 \\ 18\overline{)\,5\,8\,3\,6} \\ -\ 5\,4\downarrow \\ \hline 4\,3 \\ -\ 3\,6\downarrow \\ \hline 7\,6 \\ -\ 7\,2 \\ \hline 4 \end{array}$$

5836 ÷ 18 = 324 r 4 or $324\frac{2}{9}$

The amount of time that should be spent teaching and practising the expanded written method of long division will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.

Long Division with decimals

Method 1: Calculating with decimals

Expanded written method of long division

58·32 ÷ 18

$$\begin{array}{rl} 3\,\cdot\,2\,4 & \\ 18\overline{)\,5\,8\,\cdot\,3\,2} & \\ -\ 5\,4\,\cdot\,0\,0 & (\ \ 3 \times 18) \\ \hline {}^{3}4\,\cdot\,{}^{13}3\,2 & \\ -\ 3\,\cdot\,6\,0 & (\ 0{\cdot}2 \times 18) \\ \hline 0\,\cdot\,7\,2 & \\ -\ 0\,\cdot\,7\,2 & (0{\cdot}04 \times 18) \\ \hline 0\,\cdot\,0\,0 & \end{array}$$

Formal written method of long division

58·32 ÷ 18

$$\begin{array}{r} 3\,\cdot\,2\,4 \\ 18\overline{)\,5\,8\,\cdot\,3\,2} \\ -\ 5\,4\,\cdot\downarrow \\ \hline 4\,\cdot\,3 \\ -\ 3\,\cdot\,6\downarrow \\ \hline 0\,\cdot\,7\,2 \\ -\ 0\,\cdot\,7\,2 \\ \hline 0 \end{array}$$

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e Method 1) will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting whole and decimal numbers mentally, and in their understanding of place value.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
        3 2 4
   ┌──────────
18 ) 5 8 3 2
   − 5 4 0 0   (300 × 18)
     3 13
     4 3 2
   − 3 6 0     ( 20 × 18)
       7 2
   −   7 2     (  4 × 18)
       0
```

324 ÷ 100 = 3·24

Formal written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
        3 2 4
   ┌──────────
18 ) 5 8 3 2
   − 5 4 ↓
     4 3 |
   − 3 6 ↓
       7 2
   −   7 2
       0
```

324 ÷ 100 = 3·24

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e. Method 2) will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.